CONSTRASTING SOCIAL AND SOCIOMATHEMATICAL NORMS OF TWO GROUPS OF STUDENTS IN A POSTSECONDARY PRECALCULUS CLASS

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This paper characterizes the engagement of two groups of students in a Precalculus course at a four-year public university. A set of "Multiple Solutions Activities" was designed for the course to expose groups of students to alternative solution methods, allowing instructors to explicitly negotiate productive norms to foster students' flexible knowledge. Over the duration of the semester, the groups developed contrasting social and sociomathematical norms. One group's norms seem to be particularly influenced by students' experience taking the same course the prior semester in a more traditional format.

Keywords: Post-Secondary Education, Precalculus, Instructional Activities and Practices

Purpose

Many developmental mathematics courses, like College Algebra or Precalculus, tend to emphasize remedial content coverage and practicing procedures (Cox, 2015; Grubb, 2013; Mesa et al., 2011). This may not require students to change their mathematical practices and habits that contributed towards some students' need for further mathematical background development (Carlson et al., 2010; Goudas & Boylan, 2013). Consequently, some researchers have suggested focusing on developing students' argumentation skills and reasoning strategies (Chiaravalloti, 2009; Partanen & Kaasila, 2014).

One way to do this is to provide opportunities for students to compare, reflect, and discuss multiple solution methods (Rittle-Johnson & Star, 2007). This has been shown to help develop flexible knowledge, which Star and Rittle-Johnson (2008) characterize as the awareness of multiple problem-solving strategies and when to use them. However, students with underdeveloped mathematical skills often prefer a dependent learning style focused on mastering algorithms, making it necessary for instructors of developmental courses to negotiate productive norms, and promote mathematical practices that can help students develop flexible knowledge.

We conducted a teaching experiment in a Precalculus class at a four-year public university, in which the course instructor (first author of this paper) negotiated such productive norms and practices with the students. Specifically, the instructors attempted to aid the development of students' flexible knowledge by negotiating the social norm that it is important to understand others' work and the sociomathematical norm that an acceptable solution is one that follows any mathematically valid approach. In this paper, we analyze two groups' in-class engagement to answer the following research question: What social and sociomathematical norms developed in these groups over the semester?

Framework

Our study is framed within the emergent perspective, which views psychological and social factors as necessary to characterizing classroom activity. Continual student and teacher interactions formulate mutually established and regulated activity, which constitute norms (Cobb et al., 2001). Social norms portray the classroom participation structure, whereas sociomathematical norms are those specific to mathematical aspects of students' activity (Yackel & Cobb, 1996). Social constructs are reflexively related to psychological constructs (See Table 1). For example, as students develop

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sociomathematical norms they reorganize their mathematical values and beliefs, while productive social norms support students' positive perspectives of communal mathematical activity.

Table 1: Modified Interpretive Framework (Yackel & Cobb, 1996)

Social Perspective		Psychological Perspective
Classroom Social Norms	↔	Beliefs about one's own role, others' roles, and the general nature of mathematical activity in school
Sooiomathematical Norms	↔	Mathematical values and beliefs

Data Sources and Methods

The data were collected in a post-secondary Precalculus class of about eighty students at a four-year public university. This is the only developmental mathematics course offered by the university, and is largely populated by students who intend to major in engineering or the physical sciences, but who did not meet the pre-requisite for enrolling in Calculus. In the semester of our study, the majority of the students enrolled in the course were retaking it because they did not attain the necessary score to advance to Calculus. The instructor and teaching assistant for the course were Mathematics Education Ph.D. candidates, who previously taught this course multiple times, but not in the semester preceding this study.

Multiple Solutions Activities were designed to expose students to a variety of solution strategies, while creating opportunities for students to critique and analyze mathematical arguments. The activities were intended for groups of three or four students. Each activity had three phases. First, students solved a mathematics problem and cooperatively constructed a grading key for it. Second, students used their grading key to evaluate three fictitious students' solutions to the same problem. These solutions contained errors and/or used different approaches than those previously discussed in class (see Fig. 1-2). After analyzing these sample solutions, students were given reflection questions to compare and contrast the solutions. The last phase was a class discussion facilitated by the instructors, which helped them respond to students' concerns and bring attention to various aspects of the solutions. This allowed the instructors to model practices and explicitly negotiate norms such as: an acceptable mathematical solution may follow any mathematically valid approach and that solutions must contain explanations. Four such activities were implemented throughout the semester, and served as a key data source for the study.

Additionally, students participated in a pre- and post-course survey, which asked questions about their mathematical and role beliefs. Each item used a four point Likert scale to assess student's agreement with (1 – Disagree, to 4 – Agree) or importance of (1 – Not Important, to 4 – Very Important) given claims. For example: "The most valid ways of solving a problem are the ones discussed in class," and, "To receive full credit, my solutions must use the same methods used in class." These items aimed to assess students' openness towards other approaches, a theme the instructors advocated for to support the development of students' flexible knowledge.

Another data source was weekly writing prompts, in which students were asked to reflect on various topics such as their individual mathematical beliefs and practices. Several students were interviewed during and/or at the end of the semester to expound on their written responses.

Analysis

We analyzed video data by classifying students' utterances and activity into categories within the interpretive framework (Table 1) and coded videos in conjunction with students' written original

solutions, grading keys, and evaluations of the sample solutions. Particular attention was given to characterizing the sociomathematical norm of what constitutes an acceptable mathematical solution and the social norm of interpreting others' solutions. Similarly, students' individual responses to writing prompts were partitioned into meaning units (Tesch, 1990) and classified into categories within the interpretive framework.

In this paper, we focus on the analysis of two groups of students. Group 1 included Albert, Dwayne, Gordon, and Harry, and Group 2 was composed of Molly, Steve, Peter, and Chad (all names are pseudonyms). One of the primary reasons for choosing these two groups was that three of four students in Group 1 were taking the course for the first time whereas all of the students in Group 2 were taking the course for the second time.

The survey data were analyzed by item, using a paired t-test (JMP refers to this as a Matched Pairs test). The pre- and post-survey were paired using a non-identifying code, which students created when completing the surveys. Pre-course surveys' that did not have a matching code in the post-course survey pool, and vice versa, were not included in the analysis. In total, we analyzed 42 students' surveys, of which 26 reported taking the course in a previous semester.

Results

The quantitative analysis revealed that over the duration of the semester, in general, students developed beliefs that were supportive of developing flexible knowledge. But the qualitative analysis revealed major variations in students' perceptions, which could be seen in the norms developed in various groups of students. This was particularly evident in Group 1 and Group 2's interactions with Multiple Solutions Activities, as we will show below.

Survey Results on Flexibility

Table 2 shows the results of two survey items that assessed students' beliefs associated with flexible knowledge, both of which demonstrate a statistically significant change.

Table 2: Flexibility Survey Items and Paired T-test Results, n = 42 (1- Disagree, 4-Agree)

Question	*Pre-Mean*	*Post-Mean*	*Prob < t*
The most valid ways of solving a problem are the ones discussed in class.	2.88095	2.54762	0.0058
To receive full credit, my solution must use the same methods used in class.	2.14634	1.8297	0.0178

The decrease in mean scores suggest that students came to assign less value to following specific procedures, and view it as having less influence on receiving full credit for their work. This suggests improved openness to learning about multiple solution approaches. This change was not homogenous across all students in the class, as the next sections show.

Norms developed in Group 1

Social Norms. One of the most evident social norms that developed within this group during the Multiple Solutions Activities, was the importance of interpreting and understanding others' solutions. As the semester progressed, the students spent increasing effort to analyze the provided solutions to understand and evaluate novel approaches and to find errors in them. Even when the group to initially criticized novel approaches, this did not detract from their efforts to interpret a new method.

Another social norm that developed in this group is the importance of all group members' participation in collaboratively discussing each solution and their evaluation of it. The group exhibited a shared responsibility group to explain what they understood about each solution and to help clarify confusion to each other when possible. When analyzing novel solutions, group members would verbally share their confusion with one another. Naturally, not all group members were

uniformly vocal. To accommodate Albert's introverted demeanor, the group would often ask for his opinion on the solutions to integrate him into the group discussions. The group demonstrated that they valued each other's concerns, questions, and suggestions.

Sociomathematical Norms. We provide one vignette of Group 1's typical work that depicts the characterization of their sociomathematical norm of what constitutes an acceptable solution. During the last Multiple Solutions Activity of the semester, on the topic of inverse trigonometry, as Group 1 formed their grading rubric, they explicitly expressed awareness that there are different ways to solve the problem besides their chosen method. Harry described reluctance to form a rubric that would be limited to only one familiar way of solving:

Harry: I don't know if there is another way to solve it, so I don't want to write [grading] rules.

As they looked at the sample solutions, the group was initially dismissive of "Jennifer's" solution (Figure 2-b), which utilized right triangle trigonometry with the angle $u = sin^{-1}\left(\frac{1}{2}\right)$. This represented a novel approach that the group was unfamiliar with.

Gordon: This person is doing some weird math.
Dwayne: What did you do here? What kind of [stuff] is this? How the [heck] did you get to that?

Their lack of familiarity with her solution was obviously discomforting to them. But, despite these initial reactions, the group continued to investigate.

Gordon: [Jennifer] didn't find the inverse sine, so. They never even solved for u.
Harry: She's saying this is sine of u, this triangle, so then tangent would be opposite over adjacent, so one over one. That's what she's saying … she just didn't do it right.
Gordon: Right, because this should be one half, square root of three over two, and one (pointing to the triangle, and referring to a common right triangle).

Gordon's remark suggested that when using trigonometry, the triangle must have a hypotenuse of one. Gordon did not seem to understand how Jennifer formed her triangle. But, as Dwayne asked questions about Jennifer's approach, he was able to clarify Gordon's misconception.

Dwayne: "a" squared plus "b" squared" is "c" squared. How did [she] get two? (Pointing to the hypotenuse). Oh! [She] did one over two. That's correct though. That's just a different proportion. That is right.

This insight helped Gordon, who eventually located the exponent mistake in Jennifer's solution. After he explained the mistake to the group, he noted:

Gordon: If she did her math right, she actually would have got it, because "a" would have come out as square root of three.
Dwayne: So her process is right … but she just made one mistake. And technically her tangent work is correct for the work.

This particular example demonstrates how the social norm of collaborative analysis of an unknown solution mediated the development of sociomathematical norms within the group. This example demonstrates the group's openness to unfamiliar solutions and the sustainment of their sociomathematical norm of what constitutes an acceptable solution: a solution is acceptable if it follows any mathematically valid method. Conversely, this sociomathematical norm may have influenced the social norm of understanding other's solutions.

Norms developed in Group 2

Social Norms. All students in Group 2 had taken the course the semester prior. As the instructor tried to negotiate productive classroom social norms, this group of students developed their own set

of norms that reflected a more traditional mathematics class. One social norm that quickly developed within this group was a rejection of engagement with alternative solutions. This norm was sustained throughout the semester, as the group tried to avoid analyzing others' arguments or investigating different solutions. Instead, the group members tried to finish the activities as soon as possible, did not seek input from or ignored quiet group members. Once finished responding to the reflection questions, the group would often disengage for the rest of the class-period, including whole class discussions, which may have been particularly detrimental to the instructors' efforts of promoting students' flexible knowledge (Rittle-Johnson & Star, 2007). Since student participation in these discussions was not included in the assessment structure of the course, this may have implicitly negotiated less importance or value than other aspects of the course.

Sociomathematical Norms. The instructors attempted to negotiate the sociomathematical norm that an acceptable solution is one that follows any mathematically valid approach, not just a familiar one. However, the group was conflicted with the instructors' negotiations, instead developing an alternative norm: An acceptable solution to a problem is one that uses a familiar approach or leads to the correct answer. The following illustrates this norm.

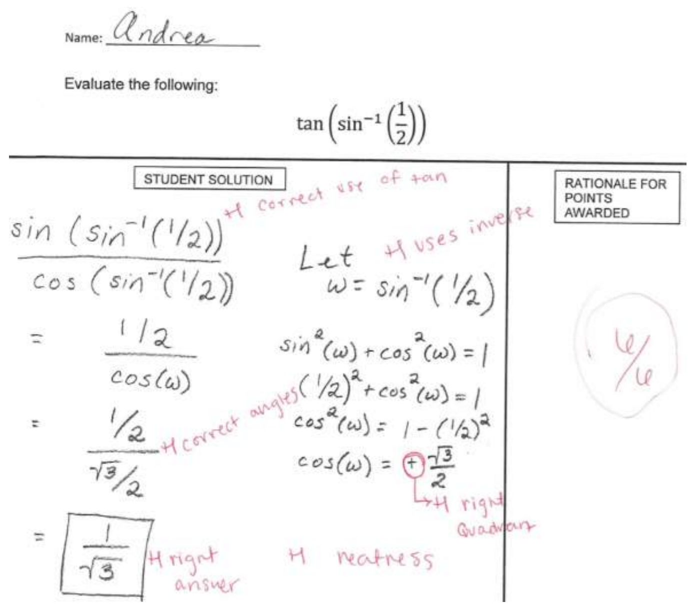
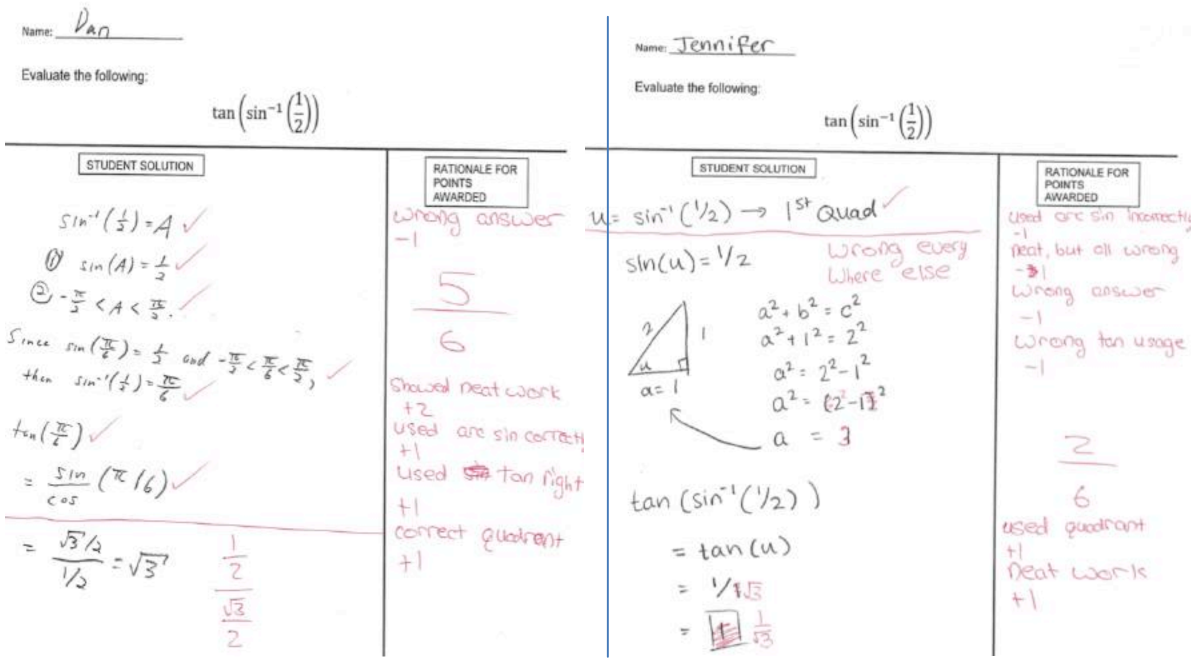


Figure 1: Molly's Grading of Andrea's Solution

During one activity, the group had to evaluate three sample solutions by the fictitious students "Andrea," "Dan," and "Jennifer," and then to compare and contrast these solutions. Andrea's solution used an unfamiliar approach but resulted in the correct answer, Dan's solution followed a method shown in class but had a wrong answer because of an intentionally included error, and Jennifer's solution was both unfamiliar and also yielded an incorrect answer.

The group favored Andrea's solution (Fig. 1), which yielded a correct answer, although it used an unfamiliar method. The group concluded that Andrea's solution was "interesting" and viable, since it "got them the right answer." The students relied on the authority of the answer to determine whether or not the approach was valid, but without thoughtful investigation.

The group was also receptive towards Dan's solution (Fig. 2-a), but for a different reason. Dan's solution resembled the approach the instructor modeled for similar problems. The approach was familiar to the group members, and eventually both Molly and Steve concluded that, "He has everything right except the answer." When students were familiar with a procedure, they were able to recognize patterns and locate errors, unlike in novel solutions like Jennifer's.



Name: Dan

Evaluate the following:

$\tan\left(\sin^{-1}\left(\frac{1}{2}\right)\right)$

STUDENT SOLUTION

$\sin^{-1}(\frac{1}{2}) = A$

① $\sin(A) = \frac{1}{2}$

② $-\frac{\pi}{2} < A < \frac{\pi}{2}$.

Since $\sin(\frac{\pi}{6}) = \frac{1}{2}$ and $-\frac{\pi}{2} < \frac{\pi}{6} < \frac{\pi}{2}$, then $\sin^{-1}(\frac{1}{2}) = \frac{\pi}{6}$

$\tan(\frac{\pi}{6})$

$= \frac{\sin}{\cos}(\pi/6)$

$= \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$

$\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$

RATIONALE FOR POINTS AWARDED

Wrong answer -1

$\frac{5}{6}$

Showed neat work +2

Used arc sin correctly +1

Used tan right +1

correct quadrant +1

Name: Jennifer

Evaluate the following:

$\tan\left(\sin^{-1}\left(\frac{1}{2}\right)\right)$

STUDENT SOLUTION

$u = \sin^{-1}(1/2) \rightarrow 1^{st}$ Quad

$\sin(u) = 1/2$

Wrong every where else

$a^2 + b^2 = c^2$

$a^2 + 1^2 = 2^2$

$a^2 = 2^2 - 1^2$

$a^2 = (2^2 - 1^2)^2$

$a = 3$

$a = 1$

$\tan(\sin^{-1}(1/2))$

$= \tan(u)$

$= 1/\sqrt{3}$

$= \frac{1}{\sqrt{3}}$

RATIONALE FOR POINTS AWARDED

Used arc sin incorrectly -1

neat, but all wrong -1

Wrong answer -1

Wrong tan usage -1

$\frac{2}{6}$

used quadrant +1

Neat work +1

Figure 2 a & b: Steve's Grading of Dan (a) and Jennifer's (b) Solutions

Jennifer's solution (Fig. 2-b) used an unfamiliar approach and resulted in an incorrect answer. The group had a scathing first response towards the solution:

> Steve: Oh God, this already looks bad. Oh yeah, this is real bad. 0 out of 6 … I hope this is not a real student, I really hope.

The only discussion in the group was to determine if Jennifer should earn points for neatness or for "getting the quadrant right." The group did not notice the arithmetic mistakes until the instructor pointed it out to them.

In general, this group did not develop the sociomathematical norms that the instructors advocated and negotiated for. Instead, they chose to focus on the correct answer, as in Andrea's solution (Fig. 1), or a familiar procedure, as in Dan's solution (Fig. 2-a). The former is indicative of intellectual hegemony, relying on authority to determine that an approach is mathematically valid, which hinders the development of students' autonomy. The group's affinity towards familiar approaches coincides with their adopted social norm of aversion to exploring novel solutions. Consequently, they were eager to discredit novel solutions. Without a source of authority or the familiarity of an approach, the group was unable to determine its mathematical validity and vehemently rejected the solution, such as Jennifer's (Fig. 2-b).

Further Differences between the Groups

The two groups described above had varying perspectives, beliefs, and practices, which may help further explain some of the differences in the norms that developed amongst them. Below are some students' written responses to the question whether they found it helpful to learn about different approaches (asked near the end of the semester).

> Dwayne (Group 1): I think it is very helpful to me … I think multiple ways of solving a problem gives me an overall better [perspective] on the problem itself and gives me a better understanding of how it is broken down.

> Albert (Group 1): It is also pretty helpful to try different things to prove it in different ways, because this will increase understanding of different methods of proving things which you may find useful in other problems.

Harry (Group 1): I find it very helpful to learn about multiple ways of solving problems. Sometimes when I see a problem approached from a different mindset I can create a mental connection between various concepts or strengthen my knowledge of how a concept works. I also like seeing how you can use seemingly unrelated math concepts to find the solution to a problem.

Molly (Group 2): No, I like to learn one set way to do the problem. The more sets and procedures there are to a problem, the more confusing it can get.

Peter and Chad (group 2) expressed that they liked learning different methods but with the intention of finding a method that was easiest for them to replicate. In general, a common theme in Group 2's responses was their preference to learn and practice through repetition, as was evident in their responses to writing prompts:

Steve: When I am learning math, I heavily rely on seeing something done out in front of me and then having myself try the example myself and try and get the same answer as the example. I will then try more examples that relate towards that problem, I just need to know the answer in the end.

Peter: I also learn from observing, and repetition … I locked myself in a study lounge and kept doing complete the square problems until it came like second nature for me, just kept repeating the steps and applying them to different problems.

Molly: Given problems to do on our own with some way, either discussion or an answer key, we are given a way to check that we are doing it right. I personally like this way because it's repetitive and that's how I learn best in math.

The differences between the two groups manifested themselves in students' attitudes towards instructor's attempts to negotiate productive norms in the course, specifically, the importance of flexible knowledge of mathematics. Group 2 expressed their frustration with the instructor's approach towards teaching the class, which differed drastically from the previous semester:

Steve: Last semester they constantly drilled in our head that there was only one way to do it.

Molly: Yeah. So that's why I feel like a lot of us, or at least personally why I'm struggling.

Steve: It's a lot different.

Molly: I don't have a set rule to follow.

These comments may represent role beliefs that reflect the expectation that the instructor is responsible for abiding by the norms of a traditional mathematics class. Despite the instructor's efforts, most of the Group 2 members' beliefs and practices remained unaffected and staunchly sustained throughout the whole semester. At the end of the semester, Steve reflected:

Steve: Throughout the semester my studying habits have not changed, I have [continued] the same strategy that I used since the beginning, but my grade has started get worse and worse, but I do not believe that it due on my part.

This comment represented a deep-rooted conflict that may explain the nature of the norms that developed in Group 2 in contrast to the instructors' negotiations and expectations.

Discussion

This study examined student engagement in a post-secondary Precalculus course, in which an instructor implemented novel instructional activities and pedagogical strategies. The course and its activities aimed towards negotiating productive social and sociomathematical norms, which were intended to support students' flexible mathematical knowledge.

Our data analysis showed that the intervention produced mixed results. At a large scale, we saw some improvement in students' placing less emphasis on a single solution strategy, and possibly more openness towards multiple solutions (Table 2). Closer examination of student engagement revealed the variability between groups of students, which was evident in social and

sociomathematical norms developed in different groups. Group 1 developed a social norm of interpreting and understanding others' solutions, which coincided with development of the sociomathematical norm that an acceptable solution may follow any mathematically valid approach. Meanwhile, Group 2's social norm of aversion to interpret or understand another's work developed concurrently with the sociomathematical norm that an acceptable solution is one with a correct answer or follows a familiar procedure. These data, along with the analysis of the differences between the two groups, suggest that there are several processes in place.

One is the interconnectedness and co-development of social and sociomathematical norms. The emergent perspective (Yackel & Cobb, 1996) emphasizes the reflexive relationships between psychological and social factors (see Table 1). In addition to these connections, our data suggest that social and sociomathematical norms may mutually influence the development of one another. For example, Group 2's sociomathematical norm of an acceptable solution as one following a familiar procedure may have influenced, and may be influenced by, the development of the social norm of avoiding engagement with non-familiar solutions. Thus, we assert that the interpretive framework can be enriched by incorporating this new dimension of reflexivity.

Second, our study demonstrates the lingering effects of detrimental classroom practices and norms. The emergent perspective describes a reflexive relationship between classroom norms and students' beliefs. Although classroom practices and norms dissolve after the conclusion of a course, the norms developed in one course affect students' individual beliefs and practices, which our study shows can persist and act as barriers to the negotiation of different norms and classroom practices in another course. This was particularly evident in Group 2's preference for repeated practice of a single procedure. Although there is value in developing procedural competences, unreflective repetitive practice may result in an illusion of competence. In our study, students seemed to hold onto inefficient practices that constrained their growth in the past, which continued to disservice them in the present. The case of Group 2 shows that changing these beliefs and negotiating productive norms, especially in developmental mathematics courses, is a gradual process. However, the case of Group 1 demonstrates the importance and positive effects of a constructive participation structure to the development of productive sociomathematical norms and improved learning outcomes.

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